INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - First Year, First Semester, 2013-14
Probability Theory-I, Midterm Examination

1. For events A, B and C defined on the same probability space, show that
(a) $P(A \cap B) \geq P(A) + P(B) - 1$, and
(b) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$. [10]

2. Suppose that there is a test for cancer with the property that 90% of those with cancer react positively wheras 5% of those without cancer also react positively. Assume that 1% of the patients in a hospital have cancer. What is the probability that a patient selected at random who reacts positively to this test actually has cancer? [8]

3. Suppose the joint probability mass function of (X, Y) is given by

$$f_{X,Y}(x,y) = \begin{cases} p^2(1-p)^y & \text{if } 0 \leq x \leq y < \infty,\ x \text{ and } y \text{ are integers;} \\ 0 & \text{otherwise,} \end{cases}$$

for $0 < p < 1$.
(a) Find the marginal probability mass functions of X and Y.
(b) Are X and Y independent?
(c) What is the name of the probability distribution of Y? [12]

4. Suppose the probability generating function of a discrete random variable Y is given by

$$Q_Y(t) = (p_1 t + 1 - p_1)(p_2 t + 1 - p_2)^2, \quad \text{for } -1 \leq t \leq 1,$$

where $0 < p_1 < 1$ and $0 < p_2 < 1$ are two fixed real numbers.
(a) Find the probability mass functions of Y.
(b) Find $E(Y)$? [10]

5. Let $X \sim \text{Poisson}(\lambda)$, and let Y be defined as

$$P(Y = y) = P(X = y | X > 0), \text{ for } y > 0.$$

(a) Find the p.m.f. f_Y of Y.
(b) Find $E(Y)$. [10]